8A Reflection

When an object is shifted from one position to another, rotated about a point, reflected over a line or enlarged by a scale factor, we say the object has been **transformed**. The names of these types of transformations are **reflection**, **translation**, **rotation** and **dilation**.

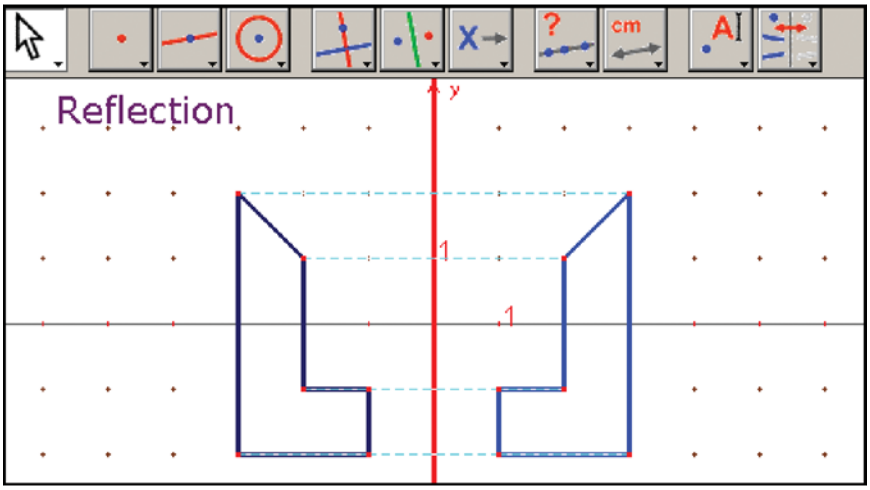
The first three of these transformations are called **isometric transformations** because the object's geometric properties are unchanged and the transformed object will be congruent to the original object. The word 'isometric' comes from the Greek words *isos* meaning 'equal' and *metron* meaning 'measure'. Dilation (or enlargement) results in a change in the size of an object to produce a 'similar' figure and this will be studied later in this chapter. The first listed transformation, reflection, can be thought of as the creation of an image over a mirror line.

Reflection creates an image reversed as in a mirror or in water.

Let's start: Visualising the image

This activity could be done individually by hand on a page, in a group using a whiteboard or using dynamic geometry projected onto a white board.

- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the Reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.



Dynamic geometry software provides a reflection tool.

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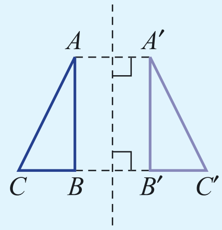
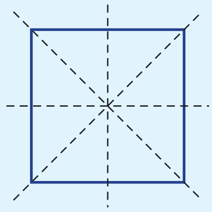
Key ideas

- **Reflection** is an **isometric transformation** in which the size of the object is unchanged.
- The **image** of a point A is denoted A'.
- Each point is reflected at right angles to the **mirror line**.
- The distance from a point A to the mirror line is equal to the distance from the image point A′ to the mirror line.
- When a line of symmetry is used as a reflection line, the image looks the same as the original figure.
- When a point is reflected across the x-axis,
  - the x-coordinate remains unchanged
  - the y-coordinate undergoes a sign change, e.g. (5, 3) becomes (5, –3); that is, the y values are equal in magnitude but opposite in sign
- When the point is reflected across the y-axis,
  - the x-coordinate undergoes a sign change
  - the y-coordinate remains unchanged, e.g. (5, 3) becomes (–5, 3)

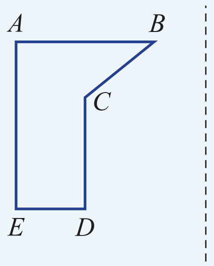




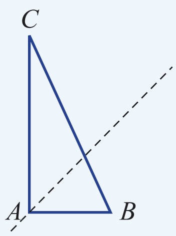
Example 1 Drawing reflected images

Copy the diagram and draw the reflected image over the given mirror line.

a

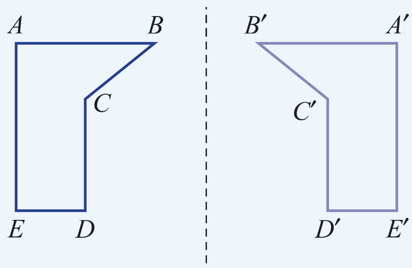


b



SOLUTION

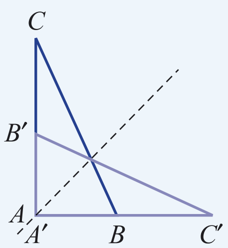
a



EXPLANATION

Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.

b



Reflect points A, B and C at right angles to the mirror line to form A', B' and C'. Note that A' is in the same position as A as it is on the mirror line. Join the image points to form the image triangle.

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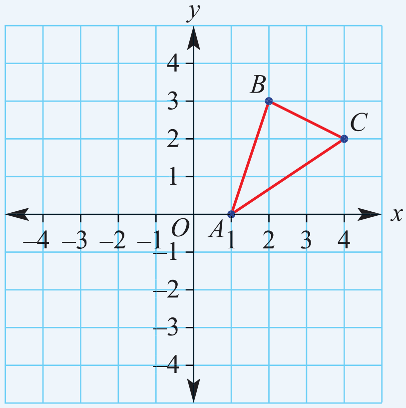
Example 2 Using coordinates

State the coordinates of the vertices A', B' and C' after this triangle is reflected in the given axes.

a x-axis **b** y-axis

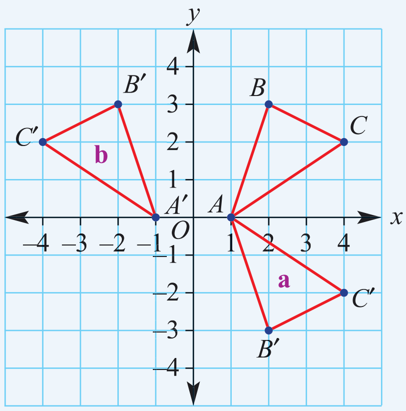


SOLUTION

a $A' = (1, 0)$
$B' = (2, -3)$
$C' = (4, -2)$

b $A' = (-1, 0)$
$B' = (-2, 3)$
$C' = (-4, 2)$

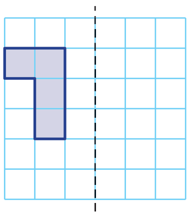
EXPLANATION



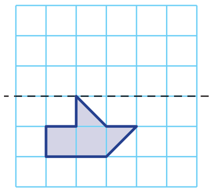
Exercise 8A

1 Use the grid to precisely reflect each shape in the given mirror line.

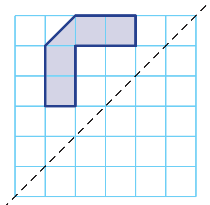
a



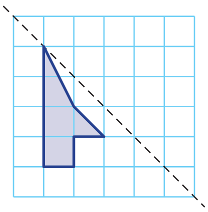
b



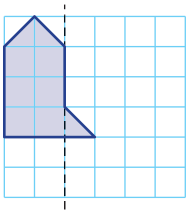
c



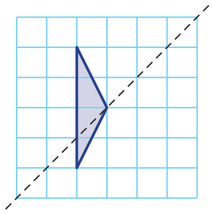
d



e



f



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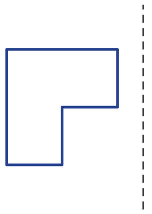
Example 1a

2 Copy the diagram and draw the reflected image over the given mirror line.

a

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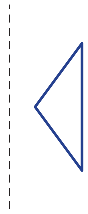
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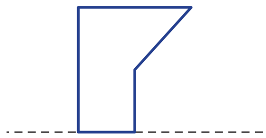
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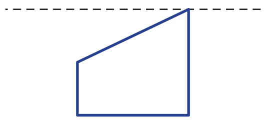
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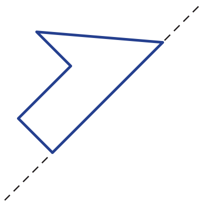
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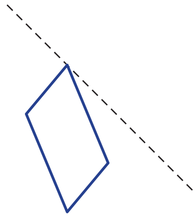
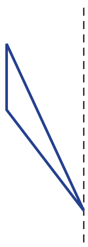


Example 1b

3 Copy the diagram and draw the reflected image over the given mirror line.

a

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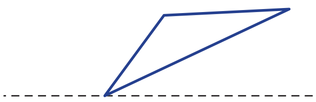
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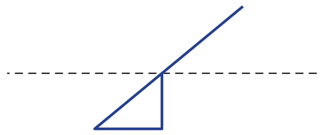
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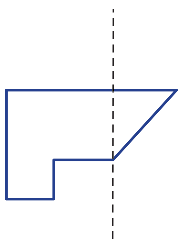
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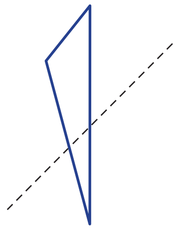
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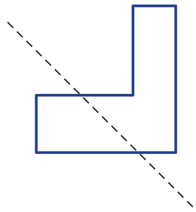
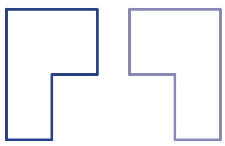


4 Copy the diagram and accurately locate and draw the mirror line. Alternatively, pencil in the line on this page.

a

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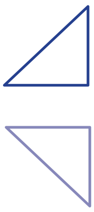
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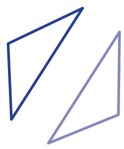
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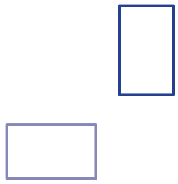
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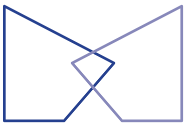
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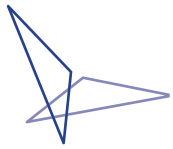

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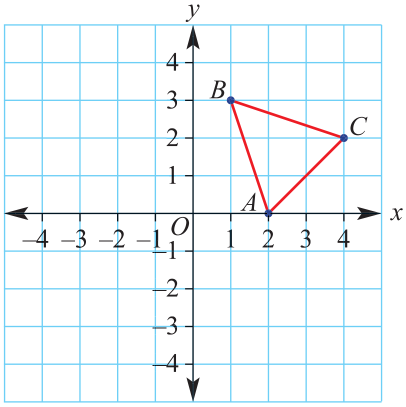
Example 2

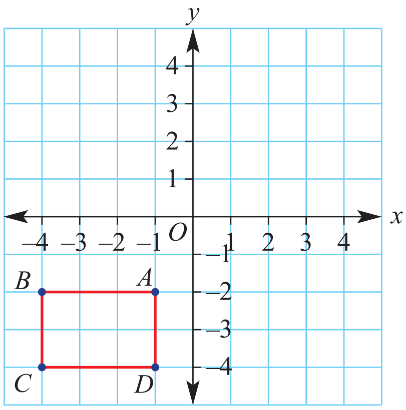
5 State the coordinates of the vertices A', B' and C' after the triangle (right) is reflected in the given axes.

a x-axis b y-axis

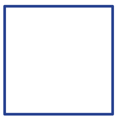


6 State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

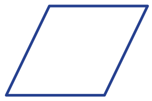
a x-axis b y-axis

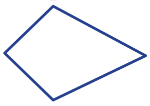


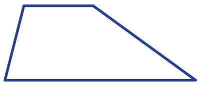
7 How many lines of symmetry do these shapes have?

a 
Square

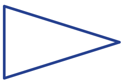
b 
Rectangle

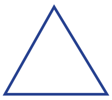
c 
Rhombus

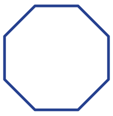
d 
Kite

e 
Trapezium

f 
Parallelogram

g 
Isosceles triangle

h 
Equilateral triangle

i 
Regular octagon

8 A point (2, 4) is reflected in the given horizontal or vertical line. State the coordinates of the image point. As an example, the graph of the mirror lines $x = 1$ and $y = -2$ are shown here.

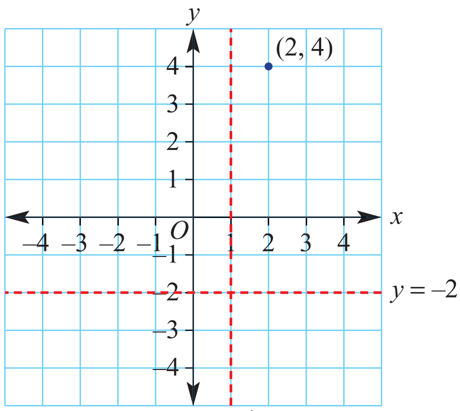
The mirror lines are:

a $x = 1$ b $x = 3$ c $x = 0$

d $x = -1$ e $x = -4$ f $x = -20$

g $y = 3$ h $y = -2$ i $y = 0$

j $y = 1$ k $y = -5$ l $y = -37$



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9 A shape with area 10 m^2 is reflected in a line. What is the area of the image shape? Give a reason for your answer.

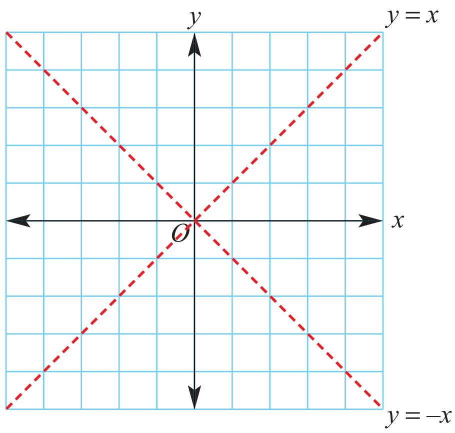
10 How many lines of symmetry does a regular polygon with n sides have? Write an expression.

11 A point is reflected in the x-axis then in the y-axis and finally in the x-axis again. What single reflection could replace all three reflections?

12 Two important lines of reflection on the coordinate plane are the line $y = x$ and the line $y = -x$ as shown.

a Draw the coordinate plane shown here. Draw a triangle with vertices $A(-1, 1)$, $B(-1, 3)$ and $C(0, 3)$. Then complete these reflections.
   i Reflect triangle ABC in the y-axis.
   ii Reflect triangle ABC in the x-axis.
   iii Reflect triangle ABC in the line $y = x$.
   iv Reflect triangle ABC in the line $y = -x$.

b Draw a coordinate plane and a rectangle with vertices $A(-2, 0)$, $B(-1, 0)$, $C(-1, -3)$ and $D(-2, -3)$. Then complete these reflections.
   i Reflect rectangle $ABCD$ in the y-axis.
   ii Reflect rectangle $ABCD$ in the x-axis.
   iii Reflect rectangle $ABCD$ in the line $y = x$.
   iv Reflect rectangle $ABCD$ in the line $y = -x$.

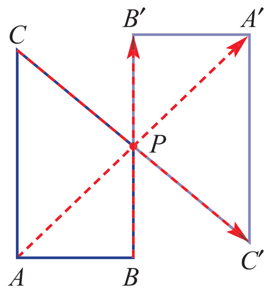


13 Points are reflected in a mirror line but do not change position. Describe the position of these points in relation to the mirror line.

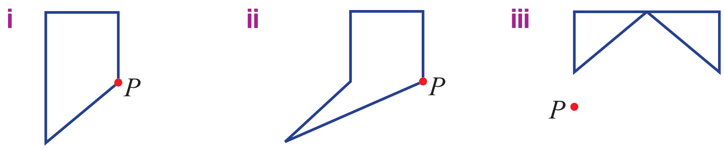
14 Research to find pictures, diagrams, cultural designs and artwork which contain line symmetry. Make a poster or display electronically.

Enrichment: Reflection through a point

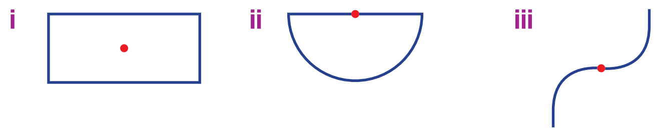
15 Rather than completing a reflection in a line, it is possible to reflect an object through a point. An example of a reflection through point P is shown here. A goes to A', B goes to B' and C goes to C' through P.



a Draw and reflect these shapes through the point P.



b Like line symmetry, shapes can have point symmetry if they can be reflected onto themselves through a point. Decide if these shapes have any point symmetry.



c How many special quadrilaterals can you name that have point symmetry?

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8B Translation

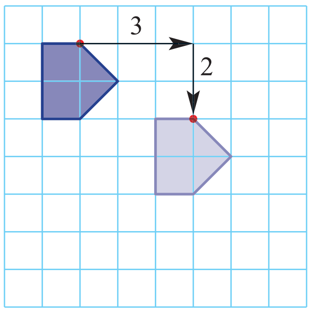
Translation is a shift of every point on an object in a given direction and by the same distance. The direction and distance is best described by the use of a translation vector. This vector describes the overall direction using a horizontal component (for shifts left and right) and a vertical component (for shifts up and down). Negative numbers are used for translations to the left and down.

Designers of animated movies translate images in many of their scenes. Computer software is used and translation vectors help to define the specific movement of the objects and characters on the screen.

Animated characters move through a series of translations.

Let's start: Which is further?

Consider this shape on the grid. The shape is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ (3 to the right and 2 down).



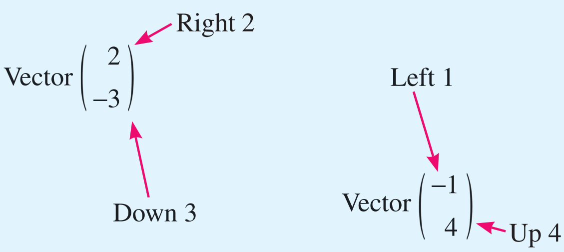
Now consider the shape being translated by these different vectors.

a $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ **c** $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ **d** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

- By drawing and looking at the image from each translation, which vector do you think takes the shape furthest from its original position?
- Is there a way that you can calculate the exact distance? How?

Key ideas

- **Translation** is an isometric transformation that involves a shift by a given distance in a given direction.
- A **vector** $\begin{pmatrix} a \\ b \end{pmatrix}$ can be used to describe the distance and direction of a translation.



  - If a is positive you shift to the right.
  - If a is negative you shift to the left.
  - If b is positive you shift up.
  - If b is negative you shift down.
- The image of a point A is denoted A'.

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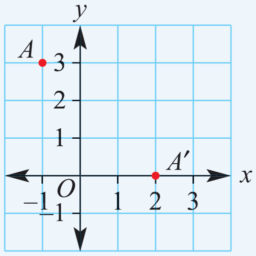
Example 3 Finding the translation vector

State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.



SOLUTION

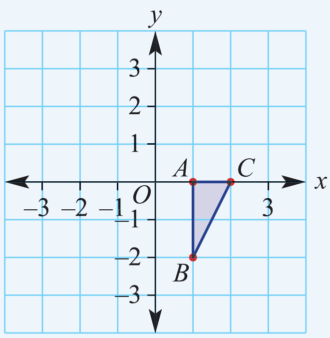
$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

EXPLANATION

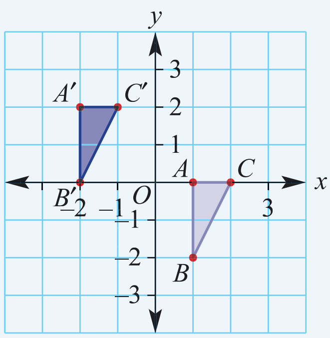
To shift A to A' move 3 units to the right and 3 units down.

Example 4 Drawing images using translation

Draw the image of the triangle ABC after a translation by the vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$.



SOLUTION



EXPLANATION

First translate each vertex, A, B and C, left 3 spaces, then up 2 spaces.

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Exercise 8B

1 Use the words left, right, up or down, to complete these sentences.

a The vector $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ means to move 2 units to the ______ and 4 units ______.

b The vector $\begin{pmatrix} -5 \\ 6 \end{pmatrix}$ means to move 5 units to the _____ and 6 units _____.

c The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ means to move 3 units to the _____ and 1 unit _____.

d The vector $\begin{pmatrix} -10 \\ -12 \end{pmatrix}$ means to move 10 units to the _____ and 12 units _____.

2 Write the vector that describes these transformations.

a 5 units to the right and 2 units down
b 2 units to the left and 6 units down
c 7 units to the left and 4 units up
d 9 units to the right and 17 units up

3 Decide if these vectors describe vertical or horizontal translation.

a $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ **b** $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$ **c** $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$ **d** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$

Example 3

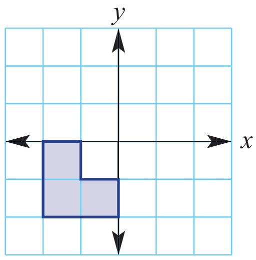
4 Write the vector that takes each point to its image. Use a grid to help you.

a $A(2, 3)$ to $A'(3, 2)$
b $B(1, 4)$ to $B'(4, 3)$
c $C(-2, 4)$ to $C'(0, 2)$
d $D(-3, 1)$ to $D'(-1, -3)$
e $E(-2, -4)$ to $E'(1, 3)$
f $F(1, -3)$ to $F'(-2, 2)$
g $G(0, 3)$ to $G'(2, 0)$
h $H(-3, 5)$ to $H'(0, 0)$
i $I(5, 2)$ to $I'(-15, 10)$
j $J(-3, -4)$ to $J'(-12, -29)$

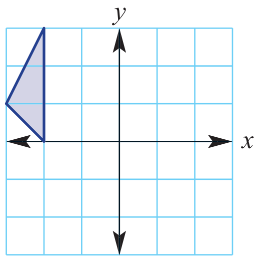
Example 4

5 Copy the diagrams and draw the image of the shapes translated by the given vectors.

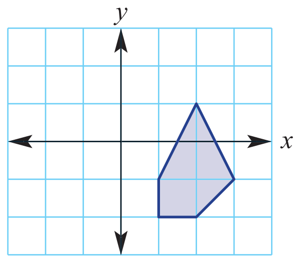
a $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$



b $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$



c $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$



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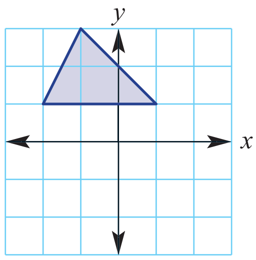
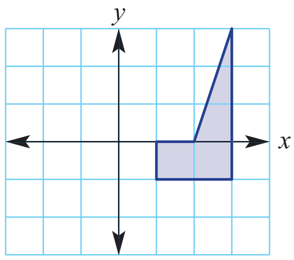
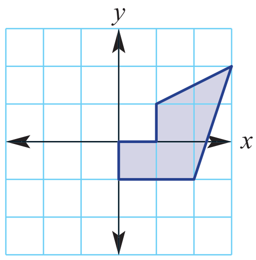
d $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$



e $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$



f $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$



6 Write the coordinates of the image of the point $A(13, -1)$ after a translation by the given vectors.

a $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$ **d** $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

e $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ **f** $\begin{pmatrix} -10 \\ -5 \end{pmatrix}$ **g** $\begin{pmatrix} -2 \\ -8 \end{pmatrix}$ **h** $\begin{pmatrix} 6 \\ -9 \end{pmatrix}$

i $\begin{pmatrix} 12 \\ -3 \end{pmatrix}$ **j** $\begin{pmatrix} -26 \\ 14 \end{pmatrix}$ **k** $\begin{pmatrix} -4 \\ 18 \end{pmatrix}$ **l** $\begin{pmatrix} -21 \\ -38 \end{pmatrix}$

---

7 Which vector from each set takes an object the greatest distance from its original position? You may need to draw diagrams to help, but you should not need to calculate distances.

a $\begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ **b** $\begin{pmatrix} -1 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

8 A car makes its way around a city street grid. A vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ represents travelling 200 m east and 300 m north.

a Find how far the car travels if it follows these vectors:
$\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

b What vector takes the car back to the origin (0, 0), assuming it started at the origin?

9 A point undergoes the following multiple translations with these given vectors. State the value of x and y of the vector that would take the image back to its original position.

a $\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}$ **b** $\begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} -7 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}$

c $\begin{pmatrix} 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ -6 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}$ **d** $\begin{pmatrix} -4 \\ 20 \end{pmatrix}, \begin{pmatrix} 12 \\ 0 \end{pmatrix}, \begin{pmatrix} -36 \\ 40 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}$

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10 A reverse vector takes a point in the reverse direction by the same distance. Write the reverse vectors of these vectors.

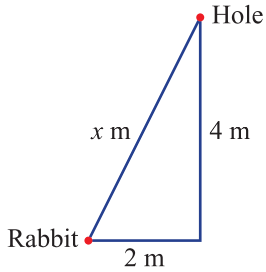
a $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **b** $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} x \\ y \end{pmatrix}$ **d** $\begin{pmatrix} -x \\ -y \end{pmatrix}$

11 These translation vectors are performed on a shape in succession (one after the other). What is a single vector that would complete all transformations for each part in one go?

a $\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -4 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 6 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ -2 \end{pmatrix}, \begin{pmatrix} -11 \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ -a \end{pmatrix}, \begin{pmatrix} -a \\ a-c \end{pmatrix}$

Enrichment: How many options for the rabbit?

12 Hunters spot a rabbit on open ground and it has 1 second to find a hole before getting into big trouble with the hunter's gun. It can run a maximum of 5 metres in one second.



a Use Pythagoras' theorem to check that the distance x m in this diagram is less than 5 m.

b The rabbit runs a distance and direction described by the translation vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$. Is the rabbit in trouble?

c The rabbit's initial position is (0, 0) and there are rabbit holes at every point that has integers as its coordinates, e.g. (2, 3) and (–4, 1). How many rabbit holes can it choose from to avoid the hunters before its 1 second is up? Draw a diagram to help illustrate your working.

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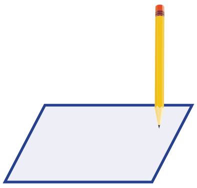
8C Rotation

When the arm of a crane moves left, right, up or down, it undergoes a rotation about a fixed point. This type of movement is a transformation called a **rotation**. The pivot point on a crane would be called the **centre of rotation** and all other points on the crane's arm move around this point by the same angle in a circular arc.

Every point on this crane's arm rotates around the pivot point when it moves.

Let's start: Parallelogram centre of rotation

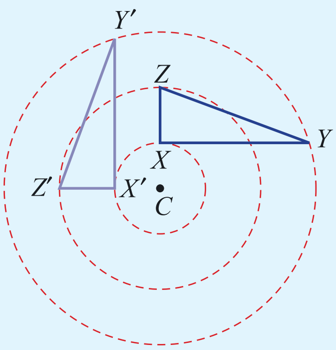
This activity will need a pencil, paper, ruler and scissors.



- Accurately draw a large parallelogram on a separate piece of paper and cut it out.
- Place the tip of a pencil at any point on the parallelogram and spin the shape around the pencil.
- At what position do you put the pencil tip to produce the largest circular arc?
- At what position do you put the pencil tip to produce the smallest circular arc?
- Can you rotate the shape by an angle of less than 360° so that it exactly covers the area of the shape in its original position? Where would you put the pencil to illustrate this?

Key ideas

- **Rotation** is an isometric transformation about a centre point and by a given angle.
- An object can be rotated clockwise or anticlockwise.
- Each point is rotated on a circular arc about the **centre of rotation** C.
  This diagram shows a 90° anticlockwise rotation about the point C.



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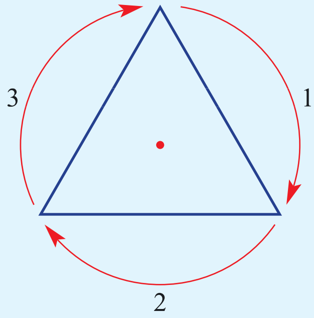
Key ideas

- A shape has **rotational symmetry** if it can be rotated about a centre point to produce an exact copy covering the entire area of the original shape.
  - The number of times the shape can make an exact copy in a 360° rotation is called the **order of rotational symmetry**. If the order of rotation is 1, then it is said that the shape has no rotational symmetry.
  - This equilateral triangle has rotational symmetry of order 3.

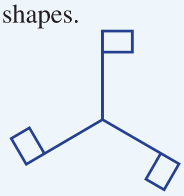


Example 5 Finding the order of rotational symmetry

Find the order of rotational symmetry for these shapes.

a

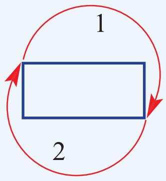
b



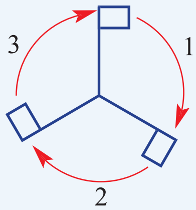
SOLUTION

EXPLANATION

a Order of rotational symmetry = 2



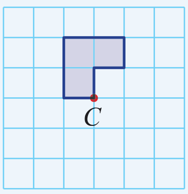
b Order of rotational symmetry = 3



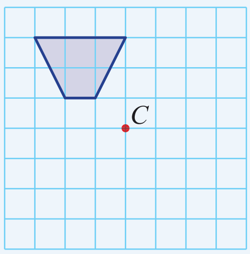
Example 6 Drawing a rotated image

Rotate these shapes about the point C by the given angle and direction.

a Clockwise by 90°



b 180°



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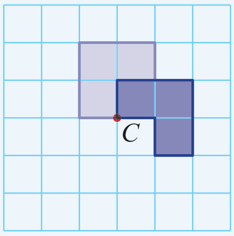
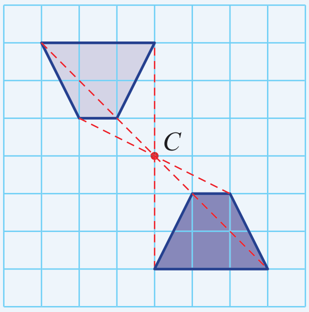
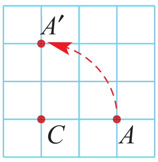
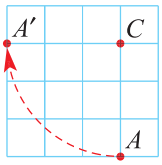
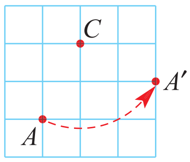
SOLUTION	EXPLANATION
a 	Take each vertex point and rotate about C by 90°, but it may be easier to visualise a rotation of some of the sides first. Horizontal sides will rotate to vertical sides in the image and vertical sides will rotate to horizontal sides in the image.
b 	You can draw a dashed line from each vertex through C to a point at an equal distance on the opposite side.

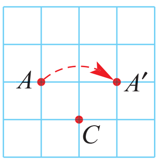
Exercise 8C

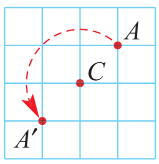
1 Point A has been rotated to its image point A'. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

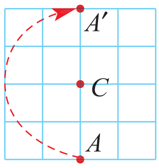
a


b


c


d


e


f


2 Complete these sentences.

a A rotation clockwise by 90° is the same as a rotation anticlockwise by _____.

b A rotation anticlockwise by 180° is the same as a rotation clockwise by _____.

c A rotation anticlockwise by _____ is the same as a rotation clockwise by 58°.

d A rotation clockwise by _____ is the same as a rotation anticlockwise by 296°.

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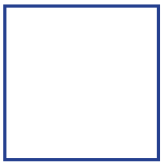
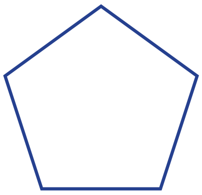
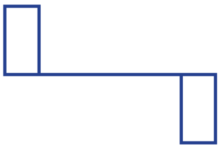
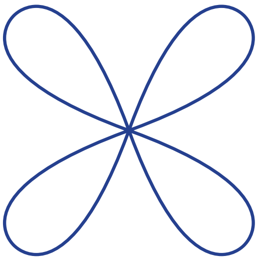
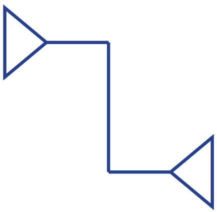
Example 5

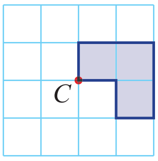
3 Find the order of rotational symmetry for these shapes.

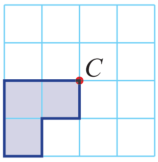
a

b

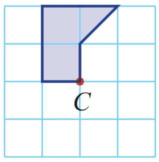
c

d

e

f


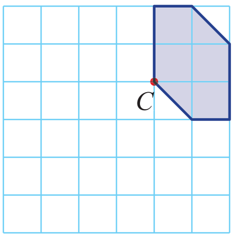
Example 6a

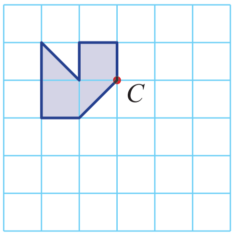
4 Rotate these shapes about the point C by the given angle and direction.

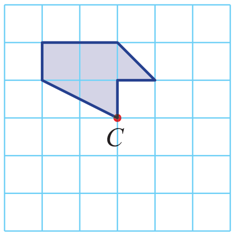
a Clockwise by 90°


b Anticlockwise by 90°


c Anticlockwise by 180°


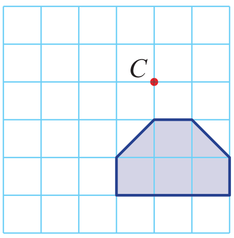
d Clockwise by 90°


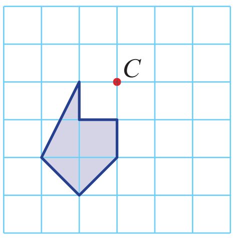
e Anticlockwise by 180°


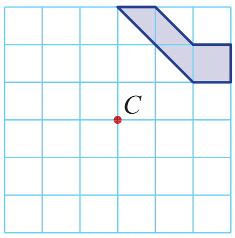
f Clockwise by 180°


Example 6b

5 Rotate these shapes about the point C by the given angle and direction.

a Clockwise by 90°


b Anticlockwise by 90°


c Anticlockwise by 180°


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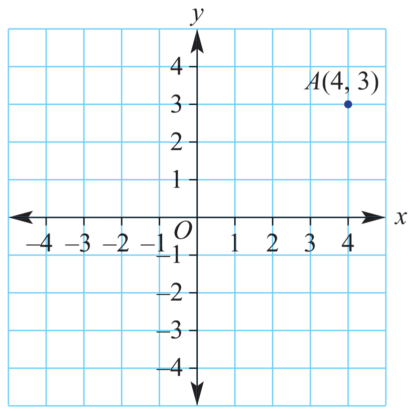
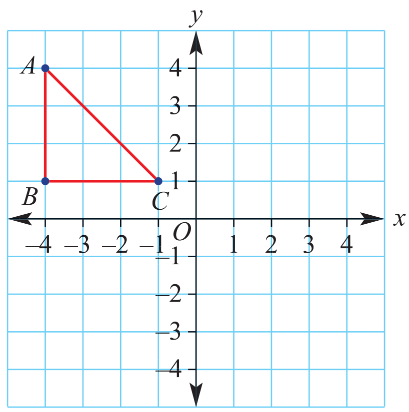
6 The point $A(4, 3)$ is rotated about the origin $C(0, 0)$ by the given angle and direction. Give the coordinates of A'.

- **a** 180° clockwise
- **b** 180° anticlockwise
- **c** 90° clockwise
- **d** 90° anticlockwise
- **e** 270° clockwise
- **f** 270° anticlockwise
- **g** 360° clockwise



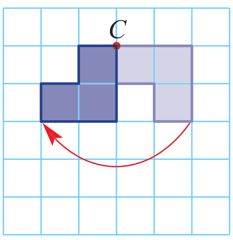
7 The triangle shown here is rotated about (0, 0) by the given angle and direction. Give the coordinates of the image points A', B' and C'.

- **a** 180° clockwise
- **b** 90° clockwise
- **c** 90° anticlockwise

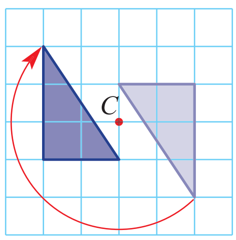


8 By how many degrees have these shapes been rotated?

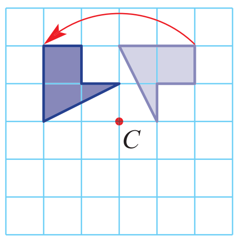
a



b



c



9 Which capital letters of the alphabet (A, B, C, …, Z) have rotational symmetry of order 2 or more?

10 Draw an example of a shape that has these properties.

- **a** Rotational symmetry of order 2 with no line symmetry
- **b** Rotational symmetry of order 6 with 6 lines of symmetry
- **c** Rotational symmetry of order 4 with no line symmetry

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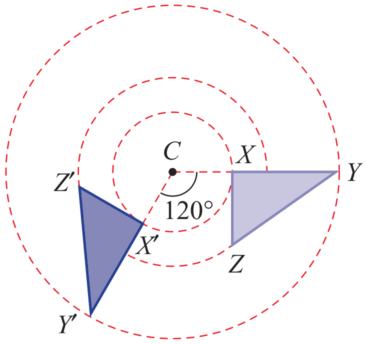
11 What value of x makes these sentences true?

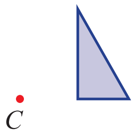
a Rotating x degrees clockwise has the same effect as rotating x degrees anticlockwise.

b Rotating x degrees clockwise has the same effect as rotating $3x$ degrees anticlockwise.

12 When working without a grid or without 90° angles, a protractor and a pair of compasses are needed to accurately draw images under rotation. This example shows a rotation of 120° about C.



a Copy this triangle with centre of rotation C onto a sheet of paper.

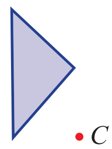


b Construct three circles with centre C and passing through the vertices of the triangle.

c Use a protractor to draw an image after these rotations.

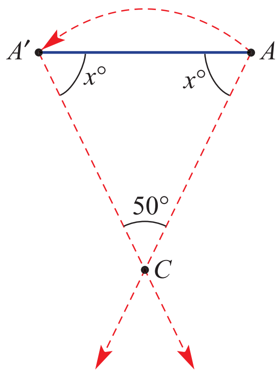
i 120° anticlockwise **ii** 100° clockwise

13 Make a copy of this diagram and rotate the shape anticlockwise by 135° around point C. You will need to use compasses and a protractor as shown in Question **12**.



Enrichment: Finding the centre of rotation

14 Finding the centre of rotation if the angle is known involves the calculation of an angle inside an isosceles triangle. For the rotation shown, the angle of rotation is 50°. The steps are given as:



i Calculate $\angle CAA'$ and $\angle CA'A$.
($2x + 50 = 180$, so $x = 65$)

ii Draw the angles $\angle AA'C$ and $\angle A'AC$ at 65° using a protractor.

iii Locate the centre of rotation C at the point of intersection of AC and $A'C$.

a On a sheet of paper draw two points A and A' about 4 cm apart. Follow the steps above to locate the centre of rotation if the angle of rotation is 40°.

b Repeat part **a** using an angle of rotation of 100°.

c When a shape is rotated and the angle is unknown, there is a special method for accurately pinpointing the centre of rotation. Research this method and illustrate the procedure using an example.

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