## Euler's formula

Resources required:
a stick of plasticine (or modelling clay).


Leonhard Euler was a Swiss philosopher, physicist and mathematician. He lived in the 18th century and he had 13 children. He claimed that he made some of his greatest mathematical discoveries while holding a baby in his arms with other children playing around his feet.

One of Euler's discoveries was the relationship between the number of faces, vertices and edges of convex polyhedra.

Take a stick of plasticine.
Roll it around in the palms of your hands to soften it.
Mould your plasticine into a cube by pressing it between a ruler and your desk. Name this cube polyhedron A.
Count the number of faces, vertices and edges of polyhedron A.
Record these numbers in the first row of the table at the top of the next page. (Leave the last column blank for now.)
Make a polyhedron with one more face by slicing off a corner of your cube with the edge of your ruler. You now have polyhedron B.
Count the number of faces, vertices and edges of polyhedron $B$ and record these numbers in the second row of the table.

Slice off one more corner. You now have polyhedron C.
Record the number of its faces, vertices and edges in the table.
Slice off one more corner. You now have polyhedron D.
Record the number of its faces, vertices and edges in the table.


Each slice adds one extra face. Each slice also adds:

- how many extra vertices?
- how many extra edges?

Activity 14-5

| Polyhedron | Number of <br> faces (F) | Number of <br> vertices (V) | Number of <br> edges(E) | F+V |
| :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |
| B |  |  |  |  |
| C |  |  |  |  |
| D |  |  |  |  |

In the last column of the table above, calculate $\mathrm{F}+\mathrm{V}$.
Compare $\mathrm{F}+\mathrm{V}$ with the values of E . What did you find?

Express your finding in an equation using the symbols $\mathrm{F}, \mathrm{V}$ and E .) Equation:
This is Euler's formula.
Fill in the table below, using the information you have already obtained about the 5 Platonic solids.

| Platonic <br> solid | NUMBER OF: |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Faces <br> (F) | Vertices <br> (V) | Edges <br> (E) | Vertices <br> on a face | Faces <br> at a vertex |
| Tetrahedron |  |  |  |  |  |
| Octahedron |  |  |  |  |  |
| Icosahedron |  |  |  |  |  |
| Cube |  |  |  |  |  |
| Dodecahedron |  |  |  |  |  |

Does Euler's formula apply to the Platonic solids?
What similarities do you notice about the regular octahedron and the cube?

What similarities do you notice about the regular icosahedron and the regular dodecahedron?

These pairs of Platonic solids are called duals.

