Making Platonic duals

Resources required

Each group of 4 students needs:

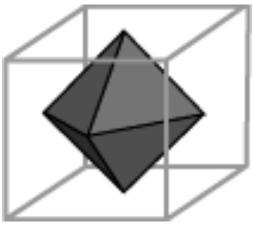
12 long pipe cleaners (at least 30 cm long) -
preferably groups of 4, 3, 3 and 2 pipe cleaners with the same colour

16 plastic drinking straws (at least 18 cm long and with no bend)

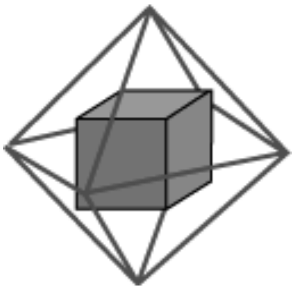
at least 5 m of thick cotton thread (eg. crotchet cotton)

a permanent marker pen

a pair of scissors.



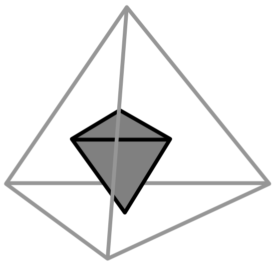
This is picture of a cube with a regular octahedron inside it.
Each vertex of the octahedron is located at the centre of one of the square faces of the cube.



This is a picture of a regular octahedron with a cube inside it.
Each vertex of the cube is located at the centre of one of the equilateral triangle faces of the regular octahedron.

A cube and a regular octahedron are called duals because their faces and vertices can occupy these complementary locations.
The faces and vertices of a regular dodecahedron and a regular icosahedron can also occupy complementary locations.

The dual of a regular tetrahedron is another regular tetrahedron.



In this activity, your group will make the following duals:

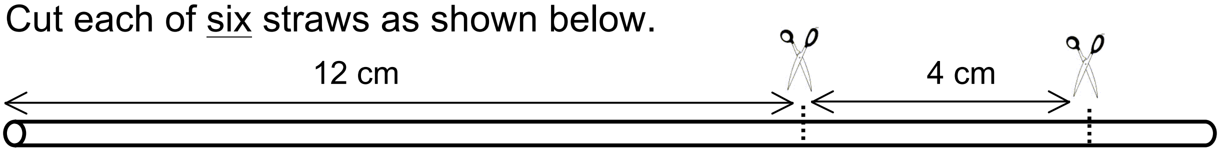
- a small tetrahedron within a large tetrahedron
- an octahedron within a cube.

There are 4 skeleton polyhedra to be made altogether.
Each polyhedron is to be made by a different person within your group.
Pairs within each group will then string their duals together.

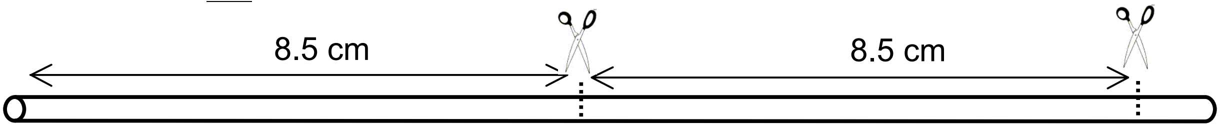
Keep your group's two dual constructions for Activity 14 - 7 "Symmetry in the Platonic solids" and Activity 14 - 9 "Molecular geometry".

To make the skeleton polyhedra:

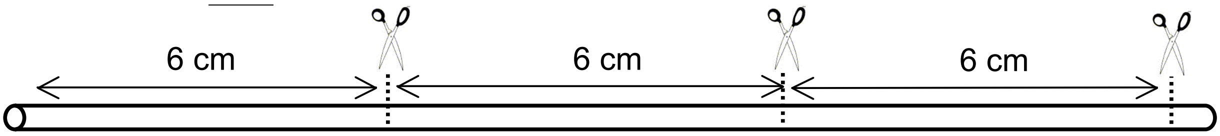
Cut each of six straws as shown below.



Cut each of six straws as shown below.



Cut each of four straws as shown below.



Follow instructions in Chapter 13 "Making skeleton solids".

The large tetrahedron requires six 12 cm straws and 3 pipe cleaners.

The small tetrahedron requires six 4 cm straws and 2 pipe cleaners.

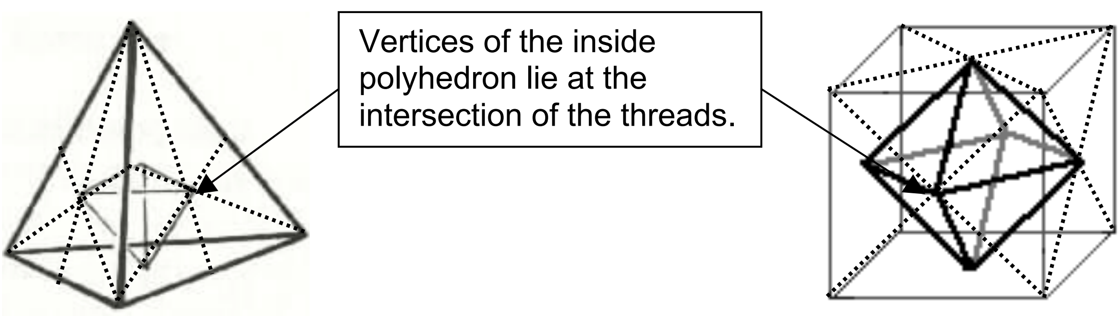
The cube requires twelve 8.5 cm straws and 4 pipe cleaners.

The octahedron requires twelve 6 cm straws and 3 pipe cleaners.

To put duals together:

Each of the vertices of the inside polyhedron is to lie at the centre of a face of the outside polyhedron.
The centres of the faces are found by bisecting each face in several directions.
These bisectors will be made with cotton thread.

The two diagrams below show the location of the thread on the visible faces of two Platonic solids.
All faces are to be bisected by your thread in the same way.



Instructions for making a tetrahedron dual

1. Measure and mark the middle of each edge of the large tetrahedron.
2. Tie thread to one vertex of the large tetrahedron.
3. Take the thread from this vertex, across a triangular face, to the middle of the opposite side. Wind it once around the middle mark.
4. Take the thread across another triangular face and wind it twice around the vertex.
5. Continue taking your thread from side to vertex to side to vertex and so on (winding it around sides and vertices as you go).
6. At some time, your thread will reach a vertex from which all the triangular faces have already been bisected. At this vertex you will need to tie and cut off your thread, then tie the thread around a vertex from which all the bisections have not yet been made.
7. When the thread has bisected all the faces 3 times (in 3 different directions), place the smaller tetrahedron, inverted, inside the larger tetrahedron so that its vertices lie where 3 threads intersect.
8. To hold the inside tetrahedron in place, tie a small piece of thread through each vertex of the small tetrahedron and the intersection of the threads on the face of the large tetrahedron.

Instructions for making a cube/octahedron dual

1. Tie thread to one vertex of the cube.
2. Take the thread from this vertex, diagonally across a square face, to the opposite vertex. Wind the thread twice around this vertex.
3. Take the thread across another square face, then wind it twice around another vertex.
4. Continue to take the thread from vertex to vertex, making diagonal bisections of square faces.
5. At some time, your thread will reach a vertex from which all the square faces have already been bisected. At this vertex you will need to tie and cut off your thread, then tie the thread around a vertex from which all the bisections have not yet been made.
6. Before you make the last diagonal bisection with your string, place your octahedron inside your cube so that its vertices lie where the threads intersect on the faces of the cube.
7. Tie a thread across the remaining face so that all the faces of the cube now have two threads bisecting them diagonally.
8. To hold the octahedron in place, tie a small piece of thread through each of its vertices and the intersection of the diagonal threads across the face of the cube.