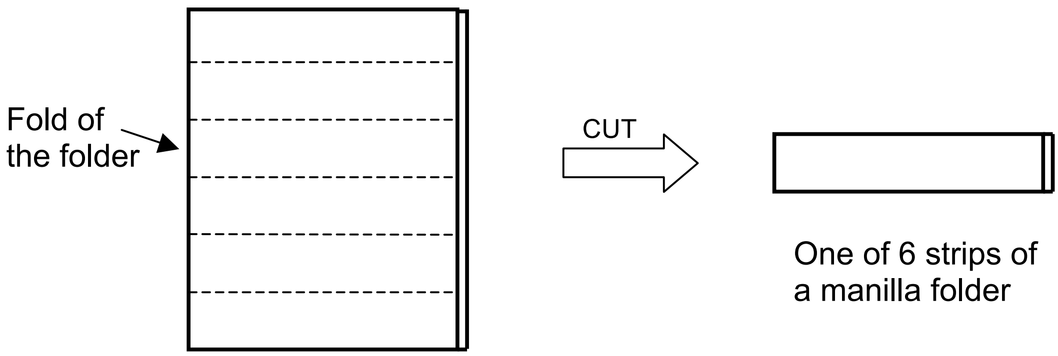
Making polyhedra from regular triangles

Resources required

Each group of 3 students needs:
a manilla folder
3 pairs of compasses
3 pairs of scissors
a stapler
sticky tape.

Your manilla folder is about 36 cm long.
With a pencil, rule 5 lines across the folder at 6 cm intervals.
Cut along each line so you have 6 strips of cardboard folded in half.



A manilla folder

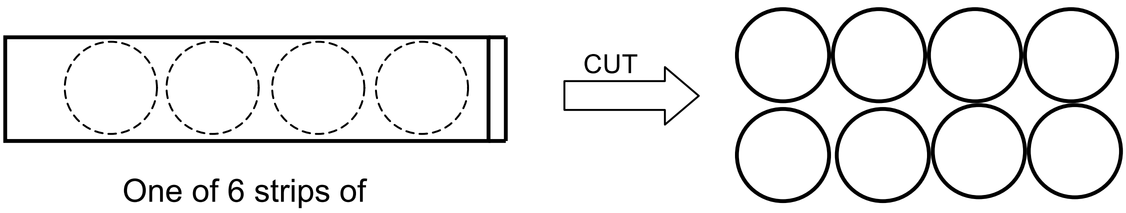
Each person in the group, take 2 of the 6 strips.

Open your compasses to a width of 2.5 cm.

Draw 4 circles along each strip, making sure that each circle has a radius of 2.5 cm.

Push your compass point through the centre of the circles (through both thicknesses of cardboard) to mark the centres.

Cut out each circle through the two thicknesses of cardboard.



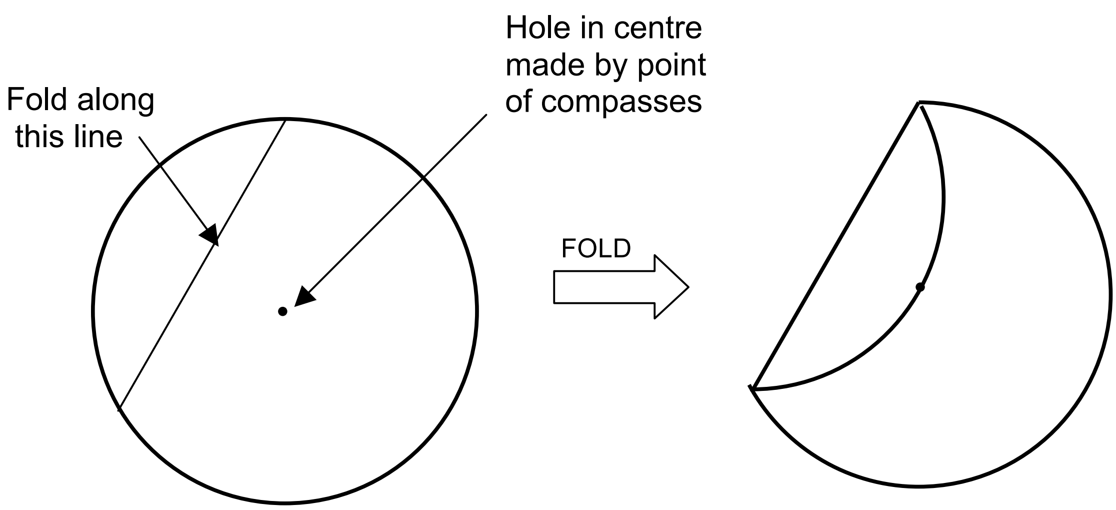
One of 6 strips of a manilla folder

Each person now has 16 cardboard circles with a diameter of 5 cm.

Combine your circles so that your group has 48 circles.

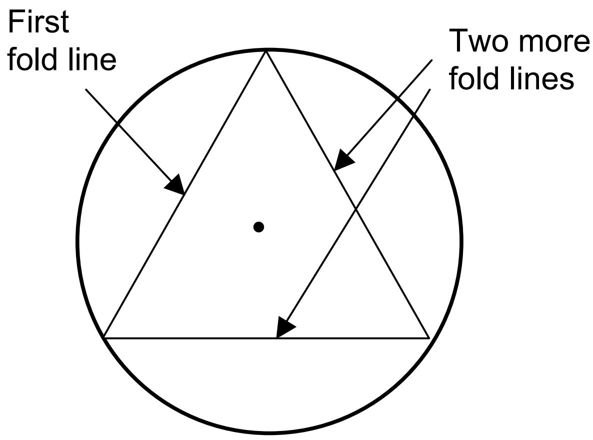
Each person take one circle and make the folds shown below.

Fold along a chord so that a point on the circumference just touches the centre of the circle.



Unfold the fold you just made.

Make 2 more fold lines in the same way, so that the circle has 3 fold lines which form a triangle (shown opposite).



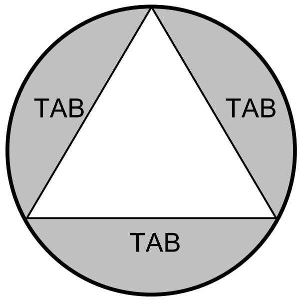
Measure the length of each of the sides of the triangle.
What type of triangle is it? ....
Is it regular or irregular? ....
Why? ....

What do you call the part of the circle that is between a chord and the circumference of that circle? ....

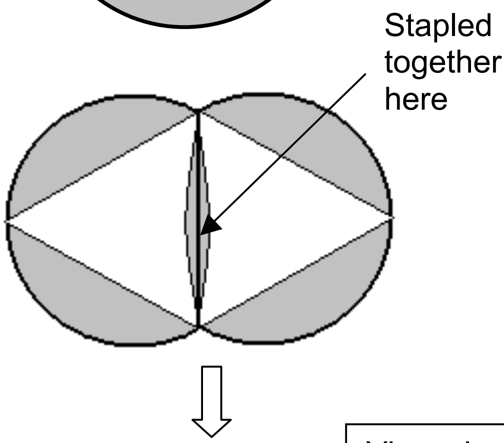
Make fold lines across the rest of your circles in the same way.

Your group should have 48 circles with 3 fold lines on them.

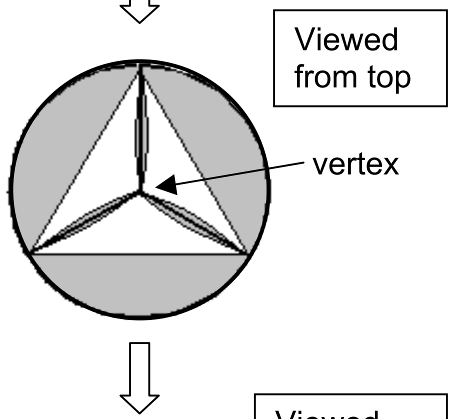
Each circle gives a regular triangle which will be used as a face. The 3 segments are 3 tabs. These will be used to join triangular faces together.



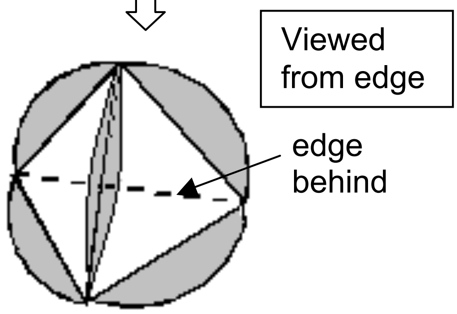
1. Place two triangles together so that one pair of edges meet each other.
Staple the pair of tabs together.
Put the staples as close as you can to the fold lines.



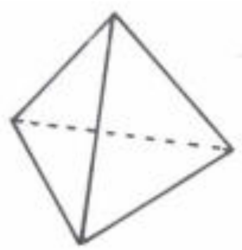
2. Take a third triangle and staple it to each of the other two (using their tabs) so that the 3 triangles meet at a common vertex.



3. Take a forth triangle and staple a tab to the free tab of each of the other three triangles.
The 4 triangles form the faces of a solid.



The solid you have made is called a regular **tetrahedron**.
"Tetra" is a Greek word meaning "four". "Hedron" means "faces",



How many faces does it have? ....

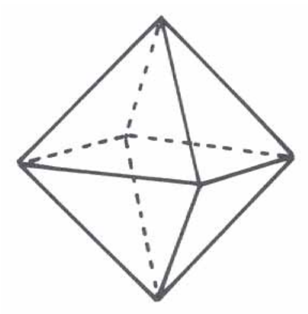
How many vertices does it have? ....

How many faces meet at each vertex? ....

How many edges does it have? ....

As a group, make another solid with your triangles.

Staple triangles together so that 4 corners meet at every vertex.
Your solid (without the tabs) should look like the one drawn below.



How many faces does it have? ....

How many vertices does it have? ....

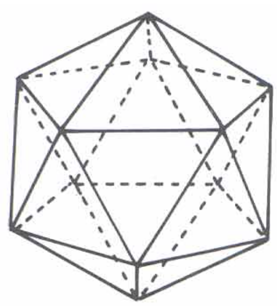
How many faces meet at each vertex? ....

How many edges does it have? ....

This solid is called a regular **octahedron**.
Why do you think it is called an octahedron?..............................

....

Now make a solid with your triangles in which 5 corners meet at every vertex.
Your solid (without the tabs) should look like the one drawn below.



How many faces does it have? ....

How many vertices does it have? ....

How many faces meet at each vertex? ....

How many edges does it have? ....

This solid is called a regular **icosahedron**.
"Icosa" is a Greek word meaning ....

With your remaining triangles, try to make a solid in which 6 corners meet at every vertex.

What happens?..............................
Why?..............................

Can any more solids be made using only regular triangles (i.e. equilateral triangles)? ....

Why or why not? ....

....

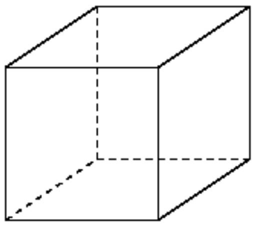
Keep the 3 regular polyhedra you have made for the next activity.

Making polyhedra from other regular polygons

Resources required:
a pair of scissors per student and access to sticky tape.

In the previous activity you made 3 regular polyhedra from 3-sided regular polygons. In this activity regular polyhedra from 4-sided regular polygons and 5-sided regular polygons are considered.

What size are the corners of a 4-sided regular polygon? ....
A solid in which 3 square faces meet at each vertex is pictured below. It is called a .... It could also be called a regular hexahedron.



How many faces does it have? ....

How many vertices does it have? ....

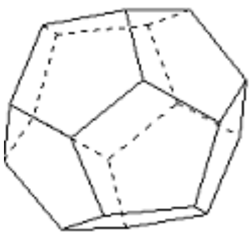
How many faces meet at each vertex? ....

How many edges does it have? ....

Could you make another regular polyhedron by putting corners of 4 squares together at every vertex? ....
Why or why not? ....

....

What size are the corners of a 5-sided regular polygon? ....
A solid in which 3 regular pentagonal faces meet at each vertex is pictured below. It is called a regular **dodecahedron**.
"Dodeca" is a Greek word meaning ....



How many faces does it have? ....

How many vertices does it have? ....

How many faces meet at each vertex? ....

How many edges does it have? ....

Could you make another regular polyhedron by putting corners of 4 regular pentagons together at every vertex? ....
Why or why not? ....

....

Why can no regular polyhedron be made with faces that are regular hexagons? ....

....

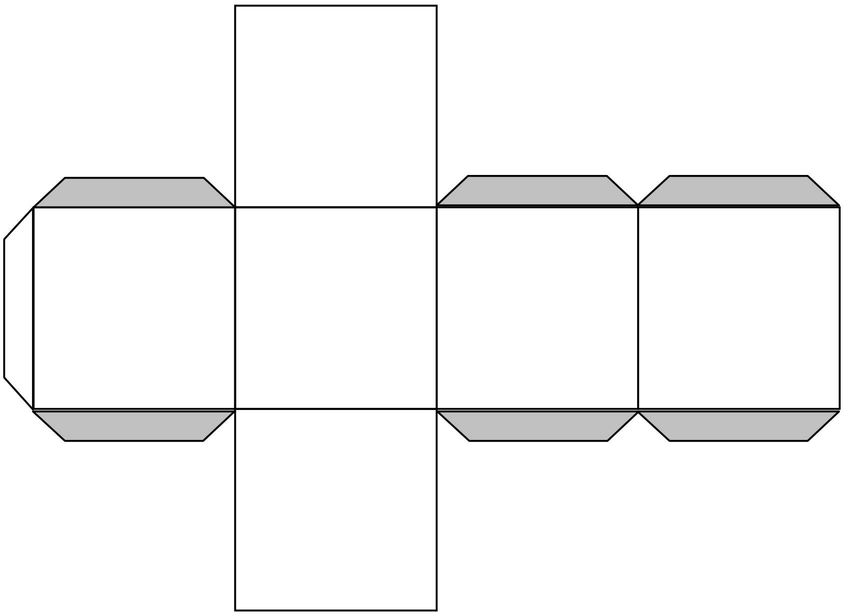
....

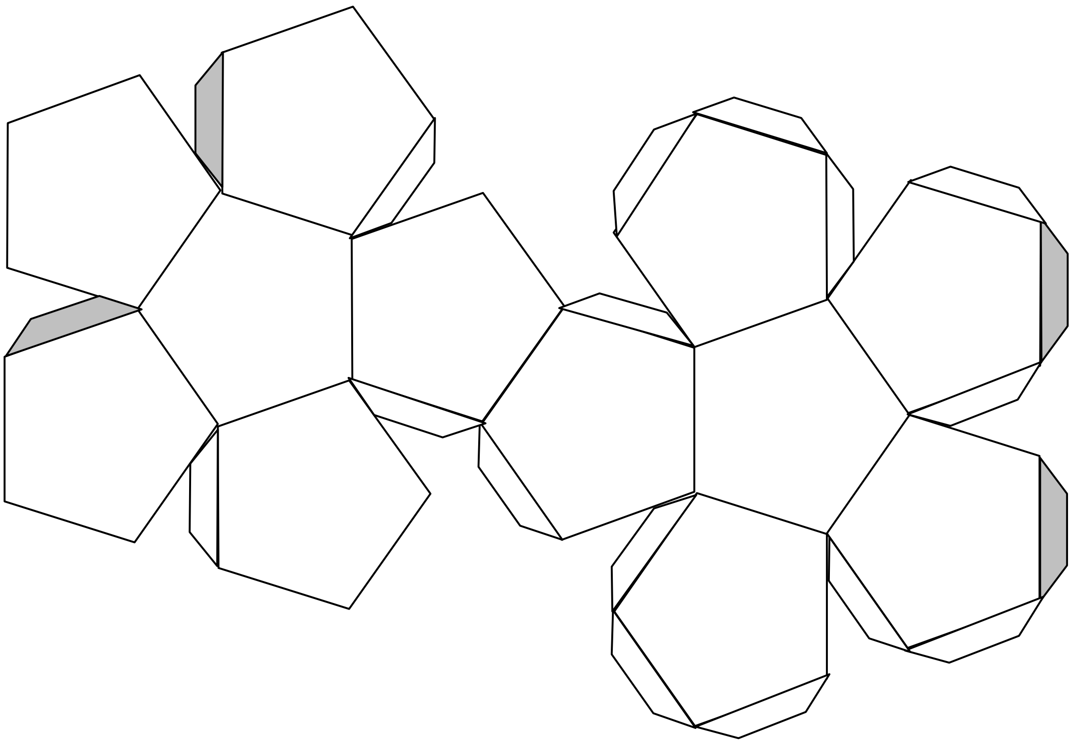
....

After you have completed the previous page, make a cube and a regular dodecahedron by cutting out the nets with tabs that are drawn at the bottom of this page. Tape the grey tabs to faces after the white tabs.

Count the faces, vertices and edges of your polyhedra to check that your answers on the previous page are right.

Colourful patterned nets (with tabs) for making these regular polyhedra as well as the three regular polyhedra with triangular faces can be downloaded from http://ccins.camosun.bc.ca/~jbritton/jbpolytess.htm.





Regular and irregular polyhedra

Resources required:
a red pen
a blue pen.

A **regular polygon**:

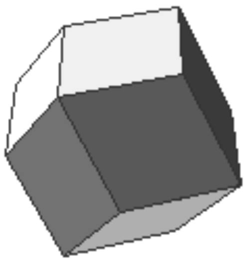
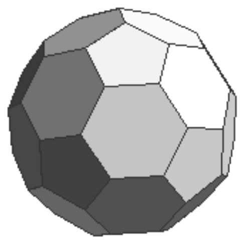
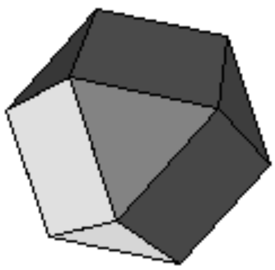
- has sides that are equal in length
- has angles that are all the same size
- has vertices that all lie on a circle called a circumcircle.

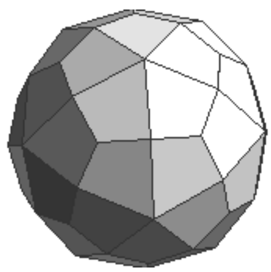
A **regular polyhedron**:

- has edges that are equal in length
- has angles that are all the same size
- has vertices that all lie on a sphere called a circumsphere.

On the pictures of polyhedra below:

- With a red pen, circle those with edges that are all equal in length.
- With a blue pen, circle those with angles that are all the same size.



The polyhedra you have circled in both red and blue, should have faces that are congruent (i.e. identical) regular polygons.

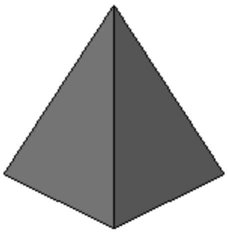
Put a cross through the polyhedra above that are not regular polyhedra.

One of the polyhedra pictured above is like a soccer ball. What shapes are the faces of a soccer ball? ....

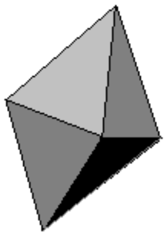
....

A regular polyhedron has vertices that all lie on a sphere. For this to happen, the same number of faces must meet at each vertex.

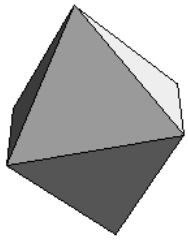
The polyhedra below all have faces that are congruent regular polygons (equilateral triangles), but one is not a regular polyhedron. Put a cross through this irregular polyhedron.



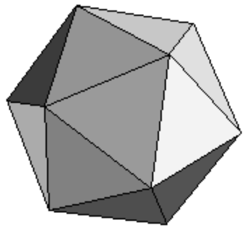
4 faces



6 faces



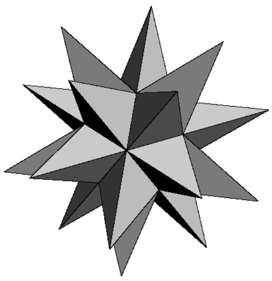
8 faces

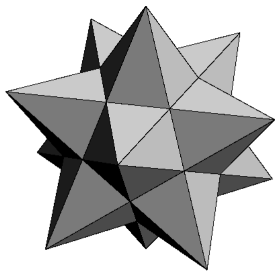


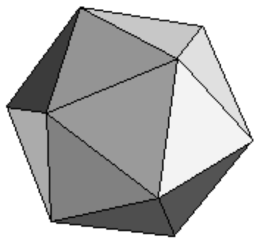
20 faces

Why is the figure that you crossed out not a regular polyhedron?

The polyhedra below all have faces that are congruent regular polygons and their vertices all lie on a sphere. Beneath each one, write whether it is convex or concave.







The concave polyhedra pictured above are called Kepler solids.
These solids are named after Johannes Kepler - a German astronomer who lived from 1571-1630.

The Kepler solids and many other polyhedra can be viewed and rotated on screen at http://mathworld.wolfram.com/topics/Polyhedra.html.

If a polyhedron is concave, it cannot have all its **dihedral angles** (i.e. the angles between its faces) equal.
Why not?

The Platonic solids

The ancient Greeks found that no more than 5 polyhedra could be made from congruent regular polygons and have vertices that are all the same.

Euclid named these solids after the ancient Greek philosopher, Plato. Plato described them in 350 BC.

The **Platonic solids** are convex regular polyhedra.
They have the following properties:

- All sides are the same length
- Faces are congruent regular polyhedra
- The same number of faces meet at each vertex
- All vertices lie on a sphere
- All dihedral angles are equal.

The ancient Greeks believed that everything in existence came from these solids. They each represented one of the five "elements": fire, water, earth, air and the universe.

Fire (dryness) was represented by the Platonic solid with the smallest volume for its surface area. The Platonic solid with the fewest vertices has the smallest volume for its surface area. This solid is the

....

Water (wetness) was represented by the Platonic solid with the largest volume for its surface area. The Platonic solid with the largest number of vertices has the greatest volume for its surface area. This solid is the

....

Earth was thought to be stand firmly (i.e. perpendicularly) on its base. This Platonic solid is the ....

The universe was represented by the Platonic solid with 12 faces because the zodiac has 12 signs. This solid is the ....

Air is mobile. It was represented by the Platonic solid that rotated most freely when held by 2 opposite vertices. This solid is the....................................